

## Problem 2.78

[Difficulty: 2]

**2.78** You intend to gently place several steel needles on the free surface of the water in a large tank. The needles come in two lengths: Some are 5 cm long, and some are 10 cm long. Needles of each length are available with diameters of 1 mm, 2.5 mm, and 5 mm. Make a prediction as to which needles, if any, will float.

**Given:** Data on size of various needles

**Find:** Which needles, if any, will float

**Solution:**

For a steel needle of length  $L$ , diameter  $D$ , density  $\rho_s$ , to float in water with surface tension  $\sigma$  and contact angle  $\theta$ , the vertical force due to surface tension must equal or exceed the weight

$$2 \cdot L \cdot \sigma \cdot \cos(\theta) \geq W = m \cdot g = \frac{\pi \cdot D^2}{4} \cdot \rho_s \cdot L \cdot g \quad \text{or} \quad D \leq \sqrt{\frac{8 \cdot \sigma \cdot \cos(\theta)}{\pi \cdot \rho_s \cdot g}}$$

From Table A.4  $\sigma = 72.8 \times 10^{-3} \cdot \frac{\text{N}}{\text{m}}$   $\theta = 0^\circ$  and for water  $\rho = 1000 \cdot \frac{\text{kg}}{\text{m}^3}$

From Table A.1, for steel  $SG = 7.83$

Hence 
$$\sqrt{\frac{8 \cdot \sigma \cdot \cos(\theta)}{\pi \cdot SG \cdot \rho \cdot g}} = \sqrt{\frac{8}{\pi \cdot 7.83} \times 72.8 \times 10^{-3} \cdot \frac{\text{N}}{\text{m}} \times \frac{\text{m}^3}{999 \cdot \text{kg}} \times \frac{\text{s}^2}{9.81 \cdot \text{m}} \times \frac{\text{kg} \cdot \text{m}}{\text{N} \cdot \text{s}^2}} = 1.55 \times 10^{-3} \cdot \text{m} = 1.55 \cdot \text{mm}$$

Hence  $D < 1.55 \text{ mm}$ . Only the 1 mm needles float (needle length is irrelevant)